1ο. Θέμα Μόρια 25

a. Μια ακτίνα φωτός εκπέμπεται με γωνία φ' σε σχέση με τον άξονα x' του συστήματος αναφοράς ενός πυραύλου που κινείται με ταχύτητα u. Δείξτε ότι η γωνία φ που σχηματίζει η διεύθυνση της ακτίνας φωτός σε σχέση με τον άξονα x του συστήματος αναφοράς του εργαστηρίου δίνεται από την εξίσωση (c=1)

$$\cos\varphi = \frac{\cos\varphi' + u}{1 + u\cos\varphi'}$$

Τώρα θεωρήστε ένα σωματίδιο σε ακινησία στο σύστημα αναφοράς του πυραύλου, το οποίο εκπέμπει ομοιόμορφα φως προς όλες τις κατευθύνσεις. Θεωρήστε ότι το 50% αυτού του φωτός το οποίο κατευθύνεται προς το μπροστινό ημισφαίριο του συστήματος αναφοράς του πυραύλου. Επίσης υποθέστε ότι ο πύραυλος κινείται πολύ γρήγορα σε σχέση με εργαστήριο. Δείξτε ότι στο σύστημα αναφοράς του εργαστηρίου αυτό το φως συγκεντρώνεται σε ένα στενό κώνο με κατεύθυνση προς τα μπροστά, του οποίου ο άξονας βρίσκεται στην διεύθυνση της κίνησης του σωματιδίου. Αυτό το φαινόμενο ονομάζεται φαινόμενο του προβολέα. (10)

b. Για κάθε μια από τις ακόλουθες περιπτώσεις γράψτε τις τέσσερις συνιστώσες της τετραορμής στη μορφή $[E, p_x, p_y, p_z]$ με c=1. Κάθε σωματίδιο έχει μάζα ηρεμίας m.

i. Ένα σωματίδιο κινείται στη θετική διεύθυνση x στο εργαστήριο με κινητική ενέργεια ίση με τρεις φορές την ενέργεια που έχει όταν είναι ακίνητο. (1)

ii. Το ίδιο σωματίδιο παρατηρείται σε έναν πύραυλο στον οποίο η παρατήρηση δείχνει ότι η κινητική ενέργεια του σωματιδίου ισούται με τη μάζα του. (1)

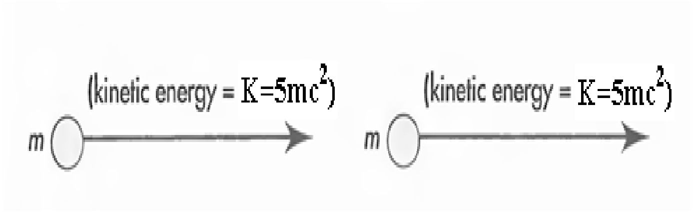
iii. Ένα άλλο σωματίδιο κινείται στη διεύθυνση y στο σύστημα του εργαστηρίου με ορμή ίση με τη διπλάσια μάζα του. (1)

iv. Ένα ακόμη σωματίδιο κινείται προς τα αρνητικά του άξονα x του εργαστηρίου με συνολική ενέργεια ίση με τέσσερις φορές τη μάζα του. (1)

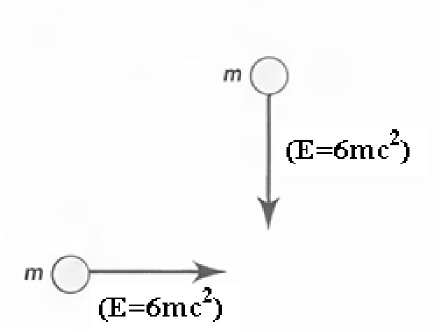
v. Επιπλέον, ένα ακόμη σωματίδιο κινείται με ίσες συνιστώσες ορμής x, y και z στο εργαστήριο και κινητική ενέργεια ίση με τέσσερις φορές την ενέργεια που έχει όταν είναι ακίνητο. (1)

c. Να υπολογιστεί η μάζα του συστήματος M_Σ για καθένα από τα συστήματα. Τα σωματίδια που αποτελούν τα συστήματα δεν αλληλεπιδρούν μεταξύ τους. Εκφράστε τη μάζα του συστήματος με όρους της μοναδιαίας μάζας m. Μη χρησιμοποιήσετε ορμές ή ταχύτητες στις απαντήσεις σας. [Σημείωση: στα παρακάτω διαγράμματα τα βέλη αντιστοιχούν στο τρισδιάστατο διάνυσμα των ορμών.]

(α)



(β)



<u>ΑΠΑΝΤΗΣΗ</u>

Α. Στο σύστημα του πυραύλου η μετατόπιση κατά x' δίνεται από την εξίσωση

$$\Delta \mathrm{x}' = \cos\varphi' \Delta \mathrm{t}'$$

Χρησιμοποιώντας τον μετασχηματισμό Lorentz

$$x = \frac{x' + ut'}{\sqrt{1-u^2}}$$

$$t = \frac{t + ux'}{\sqrt{1-u^2}}$$

Και

$$\Delta x = \frac{\Delta x' + u\Delta t'}{\sqrt{1-u^2}}$$

$$\Delta t = \frac{\Delta t + u\Delta x'}{\sqrt{1-u^2}}$$

Η ταχύτητα β της ακτίνας φωτός στο σύστημα αναφοράς είναι επίσης 1. Συνεπώς το συνημίτονο της γωνίας ανάμεσα στη διαδρομή της ακτίνας του φωτός και του άξονα x στο σύστημα αναφοράς του εργαστηρίου θα δίνεται από την έκφραση

$$\frac{\Delta x}{\Delta t} = \cos\varphi = \frac{\cos\varphi' + u}{u\ \cos\varphi' + 1}$$

Φως που κατευθύνεται προς το μπροστινό ημισφαίριο του ιδιοσυστήματος αναφοράς του πυραύλου αντιστοιχεί σε γωνίες μικρότερες από $\varphi' = 90°$. Η παραπάνω έκφραση δίνει την μέγιστη γωνία που αντιστοιχεί στο σύστημα αναφοράς του εργαστηρίου

$$\cos\varphi = u \qquad \text{για } \varphi' = 90°$$

Όλο το φως που εκπέμπεται στο μπροστινό ημισφαίριο στο σύστημα που το σωματίδιο είναι ακίνητο συγκεντρώνεται σε έναν κώνο με κατεύθυνση προς τα μπροστά με αυτό το γωνιακό άνοιγμα που παρατηρείται στο σύστημα αναφοράς του εργαστηρίου.

Β.

a. Η ολική ενέργεια του σωματιδίου ισούται με την ενέργεια που έχει όταν είναι ακίνητο m συν την κινητική ενέργεια $3\,m$. Συνεπώς $E = m + 3m = 4m$. Το σωματίδιο κινείται κατά μήκος του άξονα x, συνεπώς $p_y = p_z = 0\ και\ p_x = p$, η συνολική ορμή. Αντικαθιστούμε την τιμή του Ε στην εξίσωση $m^2 = E^2 - p^2$ για να πάρουμε

$$p^2 = E^2 - m^2 = (4m)^2 - m^2 = 16m^2 - m^2 = 15m^2$$

Συνεπώς $p_x = (15)^{1/2}m$.

Οι συνιστώσες του τετραδιανύσματος είναι

$$[E, p_x, p_y, p_z] = [4m, (15)^{\frac{1}{2}}m, 0, 0]$$

Φυσικά το μέτρο του τετραδιανύσματος ισούται με τη μάζα του σωματιδίου m - το οποίο είναι το ίδιο όποια και αν είναι η ταχύτητα, η ενέργεια ή η ορμή του.

b. Στο σύστημα αναφοράς του πυραύλου η ολική ενέργεια – ενέργεια σε ακινησία και κινητική ενέργεια – παίρνει τη τιμή $E = 2m$. Όπως προηγούμενα

$$p^2 = E^2 - m^2 = (2m)^2 - m^2 = 4m^2 - m^2 = 3m^2$$

Συνεπώς $p_x = 3^{1/2}m$ και οι συνιστώσες του τετραδιανύσματος είναι

$$[E, p_x, p_y, p_z] = [2m, 3^{\frac{1}{2}}m, 0, 0]$$

c. Σε αυτή την περίπτωση $p_x = p_z = 0$ και $p_y = p = 2m$. Επιπλέον,

$$E^2 = m^2 + p^2 = m^2 + (2m)^2 = 5m^2$$

Συνεπώς

$$[E, p_x, p_y, p_z] = [5^{\frac{1}{2}}m, 0, 2m, 0]$$

d. Μας δίνεται ότι $E = 4m$, όπως και στην περίπτωση a, με τη διαφορά ότι το σωματίδιο ταξιδεύει προς την αρνητική διεύθυνση x και άρα έχει αρνητική ορμή στη x . Συνεπώς

$$[E, p_x, p_y, p_z] = [4m, -15^{\frac{1}{2}}m, 0, 0]$$

e. Η ολική ενέργεια ισούται με $E = 5m$. Όλες οι συνιστώσες έχουν ίδια τιμή

$$p_x = p_y = p_z = p$$

Σε αυτή την περίπτωση χρησιμοποιούμε την πλήρη εξίσωση που συσχετίζει την ενέργεια, την ορμή και τη μάζα:

$$(p_x)^2 + (p_y)^2 + (p_z)^2 = 3p^2 = E^2 - m^2 = (5m)^2 - m^2 = 24m^2$$

Ή $p^2 = 8m^2$ και συνεπώς

$$[E, p_x, p_y, p_z] = [5m, 8^{1/2}m, 8^{1/2}m, 8^{1/2}m]$$

Γ. Σύστημα α: Η ενέργεια του συστήματος ισούται με την ενέργεια ηρεμίας των δύο σωματιδίων συν την κινητική ενέργεια των δύο σωματιδίων:

$$E_\Sigma = 2m + 10m = 12m$$

Το τετράγωνο της ορμής για κάθε σωματίδιο ισούται με

$$p^2 = E^2 - m^2 = (6m)^2 - m^2 = 35m^2$$

που δίνει $p = (35)^{1/2}m$. Η ορμή του συστήματος ισούται με δύο φορές το προηγούμενο: $p_\Sigma = 2\,(35)^{1/2}m$. Η μάζα του συστήματος είναι

$$M_\Sigma = [{E_\Sigma}^2 - {p_\Sigma}^2]^{1/2} = [(12m)^2 - \{2(35)^{1/2}m\}^2]^{1/2}$$

$$= [144 - 140]^{\frac{1}{2}}m = [4]^{\frac{1}{2}}m = 2m$$

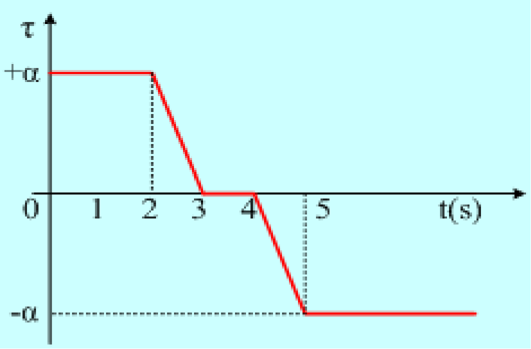
Σε αυτή την ειδική περίπτωση η μάζα του συστήματος ισούται με το άθροισμα των μαζών των αντικειμένων που συνιστούν το σύστημα. Θα μπορούσαμε να δούμε αυτό το αποτέλεσμα αμέσως παρατηρώντας το σύστημα από ένα σύστημα αναφοράς που κινείται μαζί με τα σωματίδια. Σε αυτό το σύστημα τα σωματίδια βρίσκονται σε ηρεμία και έχουν μηδενική ολική ορμή. Η ολική ενέργεια ισούται με το άθροισμα των μεμονωμένων ενεργειών ηρεμίας (τις μεμονωμένες μάζες). Συνεπώς σε αυτή την περίπτωση η μάζα του συστήματος ισούται με την ενέργειά του, που ισούται με το άθροισμα των μαζών. Επιπλέον, η μάζα του συστήματος είναι αναλλοίωτη. Συνεπώς $2m$ είναι η μάζα του συστήματος όπως υπολογίζεται σε όλα τα συστήματα αναφοράς, συμπεριλαμβανομένων και αυτό που εικονίζεται το σύστημα α.

Σύστημα β: Αυτό το μέρος του προβλήματος χρησιμοποιείται για να υπενθυμίσει ότι η ορμή είναι ένα τρισδιάστατο Ευκλείδειο διάνυσμα. Το τετράγωνο της ορμής του κάθε σωματιδίου είναι $p^2 = E^2 - m^2 = 36m^2 - m^2 = 35m^2$. Η ολική τους ορμή **δεν** ισούται με το αλγεβρικό άθροισμα των ορμών, επειδή αυτές είναι κάθετα διανύσματα. Αυτή η κάθετη διεύθυνση μας επιτρέπει να εξισώσουμε το τετράγωνο της ορμής του συστήματος με το άθροισμα των τετραγώνων των ορμών που έχει το κάθε σωματίδιο: ${p_\Sigma}^2 = 35m^2 + 35m^2 = 70m^2$. Η ενέργεια του συστήματος είναι το άθροισμα των ενεργειών (η ενέργεια είναι βαθμωτό μέγεθος και προστίθεται αλγεβρικά): $E_\Sigma = 6m + 6m = 12m$. Συνεπώς η μάζα του συστήματος είναι

$$M_\Sigma = [144m^2 - 70m^2]^{\frac{1}{2}} = [74]^{\frac{1}{2}}m = 8.602m$$

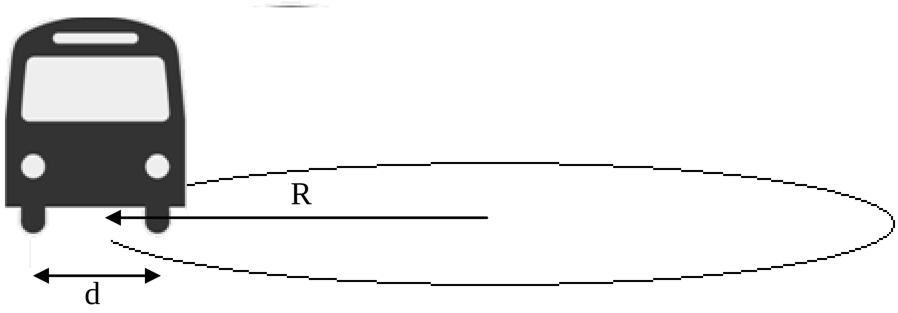
2ο. Θέμα **Μόρια 25**

a. Ένα στερεό μπορεί να στρέφεται γύρω από σταθερό άξονα και αρχικά ηρεμεί. Σε μια στιγμή δέχεται (ολική) ροπή ως προς τον άξονα, η οποία μεταβάλλεται όπως στο διάγραμμα.



Να προσδιοριστεί η γωνιακή επιτάχυνση, η γωνιακή ταχύτητα, η στροφορμή, η κινητική ενέργεια καθώς και η ισχύς της ροπής για καθένα από τα χρονικά διαστήματα 0-2, 2-3, 3-4, 4-5s Να γίνουν τα αντίστοιχα διαγράμματα. (7)

b. Ένα λεωφορείο κινείται πάνω σε μια στροφή ακτίνας καμπυλότητας R αρκετά μεγαλύτερη από τις διαστάσεις του. Βρείτε ποια συνθήκη πρέπει να ισχύει μεταξύ του συντελεστή τριβής μ, της απόστασης του κέντρου μάζας του λεωφορείου από το έδαφος h, και της απόστασης των τροχών του d ώστε αν το λεωφορείο έχει μεγάλη ταχύτητα να ολισθήσει παρά να ανατραπεί. (8)



c. Ένας δορυφόρος μάζας m κινείται σε κυκλική τροχιά γύρω από τη γη και (εκτός από τη βαρύτητα) υφίσταται μια δύναμη τριβής $\vec{F} = -k\vec{v}$.

i. Να αποδειχθεί ότι η στροφορμή του δορυφόρου ως προς το κέντρο της γης μειώνεται συνεχώς και να υπολογιστεί το μέτρο της στροφορμής ως συνάρτηση του χρόνου. (3)

ii. Να αποδειχθεί ότι η ταχύτητα του δορυφόρου αυξάνεται με το χρόνο και να υπολογιστεί η αναλυτική έκφρασή της. (3)

iii. Γενικά, τι επίδραση θα έχει η τριβή στην τροχιά του δορυφόρου; (Υπόδειξη για το τελευταίο ερώτημα: υπολογίστε την εξάρτηση της ακτίνας της τροχιάς r(t) από το χρόνο). (4)

<u>ΑΠΑΝΤΗΣΗ</u>

(a)

Η γραφική παράσταση φαίνεται στα παρακάτω σχήματα για τις παραμέτρους που φαίνονται στα σχήματα.

$\omega_0=$	0
I=	2
α=	3

t	τ	a= τ/I	$\omega_v(t)=\omega_{v-1}(t)+d\omega=\omega_{v-1}+a(t)dt$	L=Iω	$K=I\omega^2/2$	P=τω
0,00	3,00	1,50	0,00	0,00	0,00	0,00
0,50	3,00	1,50	0,75	1,50	0,56	2,25
1,00	3,00	1,50	1,500	3,00	2,25	4,50
1,50	3,00	1,50	2,250	4,50	5,06	6,75
2,00	3,00	1,50	3,000	6,00	9,00	9,00
2,50	1,50	0,75	3,565	7,13	12,71	5,35
3,00	0,00	0,00	3,750	7,50	14,06	0,00
3,50	0,00	0,00	3,750	7,50	14,06	0,00
4,00	0,00	0,00	3,750	7,50	14,06	0,00
4,50	-1,50	-0,75	3,185	6,37	10,14	-4,78
5,00	-3,00	-1,50	2,435	4,87	5,93	-7,31
5,50	-3,00	-1,50	1,685	3,37	2,84	-5,06
6,00	-3,00	-1,50	0,935	1,87	0,87	-2,81
6,50	-3,00	-1,50	0,185	0,37	0,03	-0,56
7,00	-3,00	-1,50	-0,565	-1,13	0,32	1,70

$$\omega(t)=\omega_0+1,5\int_{t_0}^{t}(1-t)dt=\omega_0+1,5(t-\frac{t^2}{2})\Big|_{t_0}^{t}$$

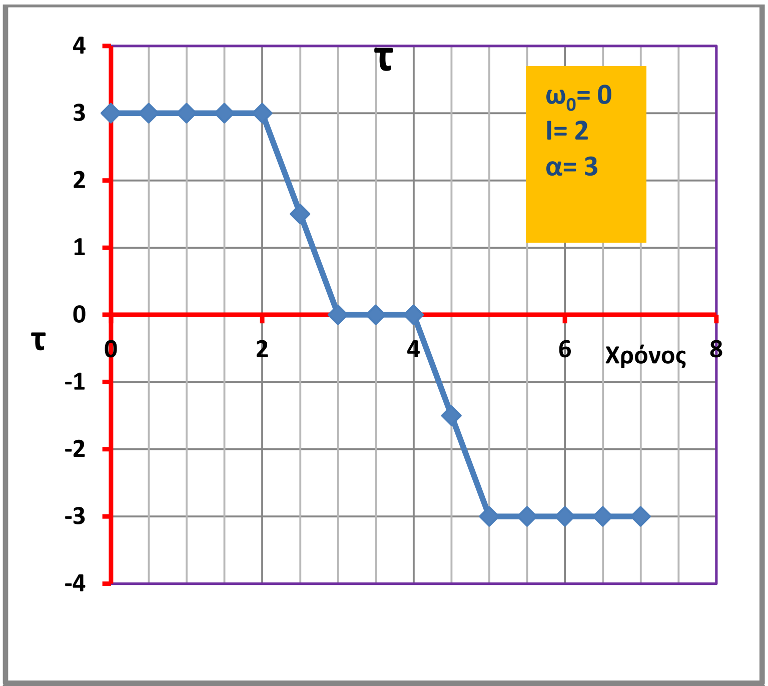
$$\omega(2,5)=3,0+1,5(t-\frac{t^2}{2})\Big|_{0}^{0,5}=3,565$$

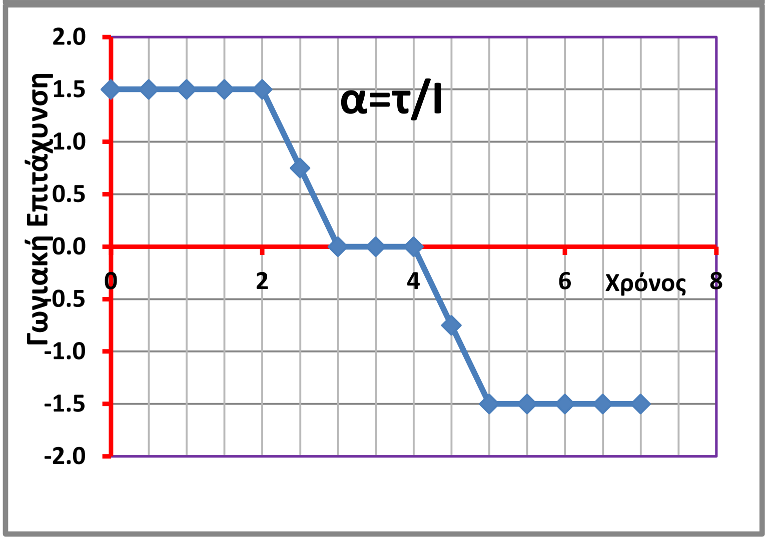
$$\omega(3)=3,0+1,5(t-\frac{t^2}{2})\Big|_{0}^{1}=3,75$$

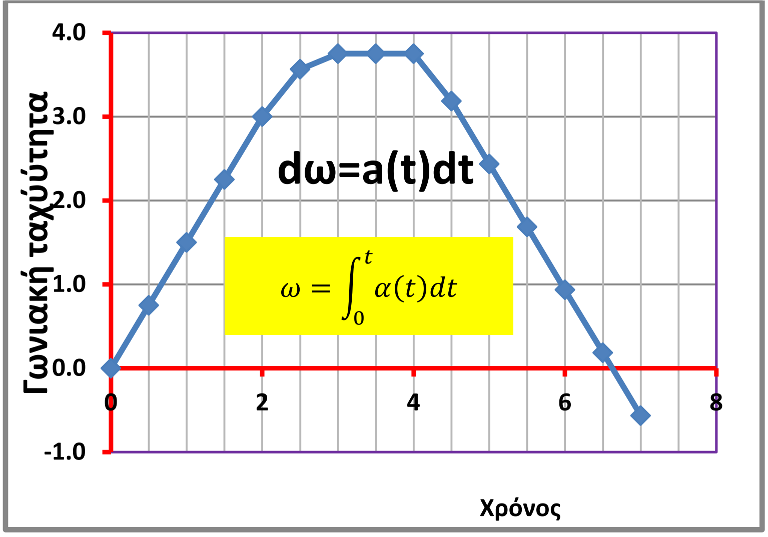
$$\omega(t)=\omega_0-1,5\int_{t_0}^{t}(1-t)dt=\omega_0-1,5(t-\frac{t^2}{2})\Big|_{t_0}^{t}$$

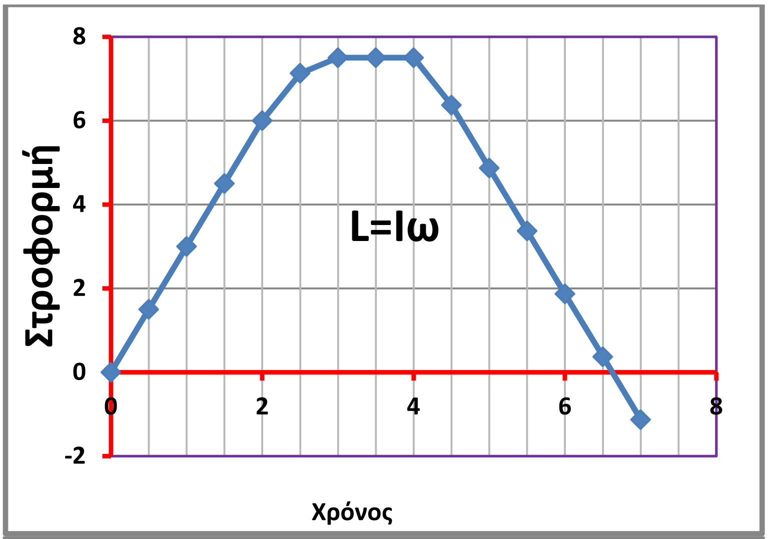
$$\omega(4,5)=3,75-1,5(t-\frac{t^2}{2})\Big|_{0}^{0,5}=3,185$$

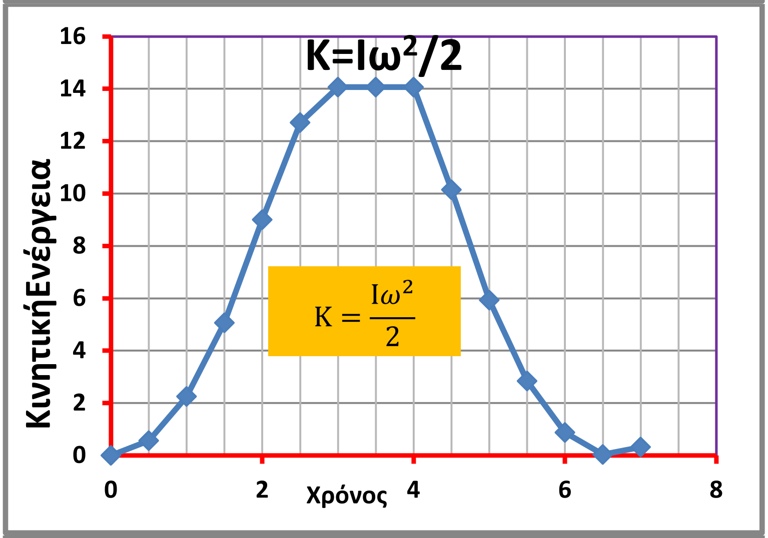
$$\omega(5)=3,75-1,5(t-\frac{t^2}{2})\Big|_{0}^{1}=2,435$$

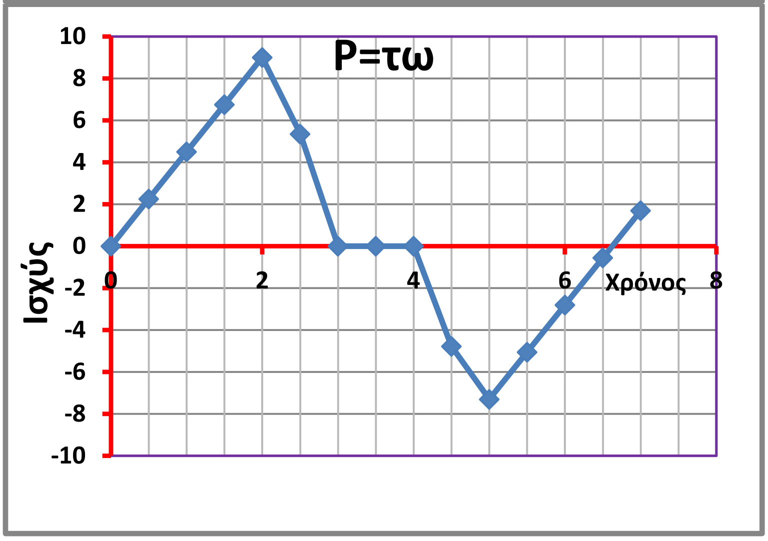

τ
ω_0= 0
I= 2
α= 3
τ
Χρόνος
4
3
2
1
0
-1
-2
-3
-4
0
2
4
6
8


α=τ/I
Γωνιακή Επιτάχυνση
Χρόνος
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0
2
4
6
8


dω=a(t)dt
$\omega = \int_0^t \alpha(t)dt$
Γωνιακή ταχύτητα
Χρόνος
4.0
3.0
2.0
1.0
0.0
-1.0
0
2
4
6
8







Παρατηρείστε ότι η γωνιακή επιτάχυνση έχει την "ίδια" εξάρτηση (=εκτός από μια σταθερά) με την ροπή, η γωνιακή ταχύτητα με τη στροφορμή και περίπου με την κινητική ενέργεια.

(b) Η συνολική δύναμη λόγω της τριβής είναι $F = \mu mg$ όπου m η μάζα του λεωφορείου. Η δύναμη αυτή είναι η κεντρομόλος δύναμη $m\frac{\upsilon^2}{R}$ και προκειμένου να ολισθήσει θα πρέπει $m\frac{\upsilon^2}{R} > \mu mg \Rightarrow \frac{\upsilon^2}{gR} > \mu$ Το λεωφορείο θα ανατραπεί όταν η ροπή της φυγοκέντρου είναι μεγαλύτερη από τη ροπή του βάρους ως προς άξονα που διέρχεται από τις εξωτερικές ρόδες του λεωφορείου δηλαδή όταν $m\frac{\upsilon^2}{R}h > mg\left(\frac{d}{2}\right) \Rightarrow \frac{\upsilon^2}{gR} > \frac{d}{2h}$ Επομένως θα πρέπει να ισχύει

$$\mu < \frac{d}{2h}$$

(i) Η στροφορμή $\vec{L} = \vec{r} \times m\vec{v} = mvr\hat{z},$ όπου $\left|\vec{L}\right| = mvr,$ (1)

Για να υπάρξει μεταβολή της στροφορμής πρέπει να επιδρά στον δορυφόρο μια ροπή. Η μόνη δύναμη που εμφανίζει μη μηδενική ροπή (ως προς το κέντρο περιστροφής) είναι η δύναμη τριβής $\vec{F} = -k\vec{v}$ (η δύναμη της παγκόσμιας έλξης, $\vec{F}_B = -\frac{GmM}{r^2}\hat{r}$, δεν δίνει ροπή ως προς το κέντρο της γης επειδή είναι συγγραμική με το $\vec{r}$).

Η μεταβολή της στροφορμής: $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \Rightarrow \left|\frac{d\vec{L}}{dt}\right| = rF = -rkv$ (αφού $\vec{r} \perp \vec{v}$)

άρα $$\frac{dL}{dt} = -rkv \quad (2)$$

Από τις σχέσεις (1) και (2) έχουμε την παρακάτω σχέση (3):

$$\frac{dL}{d} = -\underset{t}{k}\frac{L}{m} \Rightarrow \frac{dL}{L} = -\frac{k}{m}dt \Rightarrow \int_{L_0}^{L}\frac{dL}{L} = -\frac{k}{m}t \Rightarrow \ln L - \ln L_0 = -\frac{k}{m}t \Rightarrow L = L_0 e^{-\frac{k}{m}t} = mv_0 r_0 e^{-\frac{k}{m}t}$$

(ii) Η δύναμη της παγκόσμιας έλξης είναι η κεντρομόλος της κυκλικής τροχιάς, δηλαδή,

$$\frac{GmM}{r^2} = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}} \Rightarrow v = \sqrt{\frac{GmMv}{L}} \Rightarrow v = \frac{GmM}{L} \Rightarrow v = \frac{GmM}{L_0}e^{\frac{k}{m}t}, \quad (4)$$

άρα η ταχύτητα του δορυφόρου αυξάνεται με το χρόνο.

(iii) Από τις σχέσεις (1) έως (4) $\Rightarrow$ $r(t) = \frac{v_0^2 r_0^2}{GM}e^{-2\frac{k}{m}t}$, η τριβή προκαλεί την εκθετική μείωση της ακτίνας με μεγαλύτερο ρυθμό από τον ρυθμό αύξησης της ταχύτητας.

3ο. Θέμα

Μόρια 25

a. Γράψτε μέχρι δύο σελίδες για την σημασία της μετάπτωσης του άξονα και της κίνησης της Γης γύρω από τον Ήλιο και τα πιθανά αποτελέσματα αυτής στο κλίμα καθώς και στην εξέλιξη των ειδών σύμφωνα με την θεωρία του Milanovitch. (βλ. δημοσίευση που έχει αναρτηθεί στο διαδίκτυο.) (6)

b. Κύλινδρος ακτίνας R και μάζας m κυλίεται χωρίς να ολισθαίνει προς τα κάτω σε πλαγιά υπό κλίση, διατηρώντας τον άξονά του οριζόντιο.

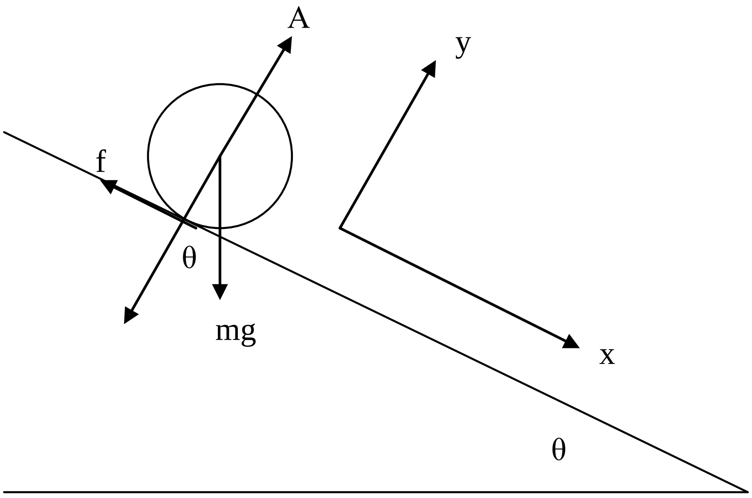
  i. Υπολογίστε την επιτάχυνσή του, αν δίνεται ότι η ροπή αδράνειας ως προς τον άξονά του είναι ${}^{1}\!/_{2}\,mR^2$. (6)

  ii. Αν κατά την κίνησή του προσκολλώνται ομοιόμορφα σε ολόκληρη την επιφάνεια του κυλίνδρου σωματίδια ίδιας πυκνότητας μάζας με του κυλίνδρου, με αποτέλεσμα την αύξηση της ακτίνας με σταθερό ρυθμό $\left({}^{dR}\!/_{dt} = \lambda\right)$, γράψτε τις εξισώσεις της κίνησης και βρείτε μια σχέση ανάμεσα στην επιτάχυνση και την γωνιακή ταχύτητα του κυλίνδρου. (13)

<u>ΑΠΑΝΤΗΣΗ</u>

<u>Λύση</u>

A) Π.χ. W.S. Broecker and G. Denton στο Scientific American, σελ. 49, Ιανουάριος 1990.



B) **(i)** Στο διπλανό σχήμα παρουσιάζονται οι δυνάμεις, η αντίδραση του εδάφους A, η τριβή f και το βάρος mg. Αναλύω τις δυνάμεις στις δύο διευθύνσεις του σχήματος και έχουμε τις εξισώσεις

Κατά τον Y: $A = mg\cos\theta$

Κατά τον X: $mg\sin\theta - f = m\gamma$

Αφού έχουμε μόνο κύλιση και όχι ολίσθηση,

$$\upsilon = \omega R \quad \Rightarrow \quad \gamma = R\frac{d\omega}{dt}$$

Από τις ροπές ως προς το κέντρο του κυλίνδρου $Rf = I\frac{d\omega}{dt} \quad \Rightarrow \quad f = \frac{I}{R^2}\gamma$

Ο συνδυασμός των παραπάνω οδηγεί στην σχέση $\gamma = \dfrac{g\sin\theta}{m\left(1+\dfrac{I}{mR^2}\right)}$, που για $I = {}^{1}\!/_{2}\, mR^2$ δίνει την τιμή $\gamma = \dfrac{2g\sin\theta}{3}$.

(ii) Αν η ακτίνα μεταβάλλεται με ρυθμό ${}^{dR}\!/_{dt} = \lambda$, τότε ο κύλινδρος πυκνότητας ρ και μήκους L θα υφίσταται μια μεταβολή της μάζας του $m = \pi R^2 L\rho$ με ρυθμό $\dfrac{dm}{dt} = (2\pi RL)\rho\dfrac{dR}{dt} = \dfrac{2m\lambda}{R}$.

Η αντίστοιχη μεταβολή της ροπής αδράνειας $I = {}^{1}\!/_{2}\, mR^2 = {}^{1}\!/_{2}\, \pi R^4 L\rho$ θα είναι ίση προς $\dfrac{dI}{dt} = 2\pi\rho LR^3\dfrac{dR}{dt} = 2Rm\lambda$

Η σχέση της ταχύτητας κατά την διεύθυνση του άξονα x με την γωνιακή ταχύτητα θα δίνει ότι $\upsilon = \omega R$ και επί πλέον ότι $\gamma = \dfrac{d\upsilon}{dt} = \omega\lambda + R\dfrac{d\omega}{dt}$.

Η σχέση για τις δυνάμεις κατά τον άξονα x θα πάρει την μορφή

$$mg\sin\theta - f = \frac{dp}{dt} = m\gamma + \frac{dm}{dt}\upsilon = m\gamma + \frac{2m\lambda}{R}\upsilon = m\gamma + 2m\lambda\omega$$

Οπότε $f = mg\sin\theta - m\left(\omega\lambda + R\dfrac{d\omega}{dt}\right) - 2m\lambda\omega = mg\sin\theta - 3m\lambda\omega - mR\dfrac{d\omega}{dt}$

Ενώ η σχέση για την στροφορμή θα γίνει

$$Rf = \frac{dL}{dt} = \frac{d(I\omega)}{dt} = \frac{1}{2}mR^2\frac{d\omega}{dt} + 2Rm\lambda\omega \quad \Rightarrow \quad f = \frac{1}{2}mR\frac{d\omega}{dt} + 2m\lambda\omega$$

Ο συνδυασμός των εξισώσεων δίνει την σχέση

$$f = mg\sin\theta - 3m\lambda\omega - mR\frac{d\omega}{dt} = \frac{1}{2}mR\frac{d\omega}{dt} + 2m\lambda\omega$$

Δηλαδή την σχέση

$$g\sin\theta = \frac{3}{2}R\frac{d\omega}{dt} + 5\lambda\omega$$

Η λύση προκύπτει από την προηγούμενη εξίσωση, δηλαδή

$$\frac{2}{3}g\sin\theta = R\frac{d\omega}{dt} + \frac{10}{3}\lambda\omega \quad \Rightarrow \quad \frac{2}{3}gR^{7/3}\sin\theta = R^{10/3}\frac{d\omega}{dt} + \frac{10}{3}R^{7/3}\omega\frac{dR}{d\omega} = \frac{d}{dt}\left[R^{10/3}\omega\right]$$

Και από την σχέση $R = R_o + \lambda t$ θα έχουμε ότι

$$R^{10/3}\omega = \frac{2}{3}g\sin\theta\int R^{7/3}d \;= \frac{2}{3}t g\sin\theta\int (R_o + \lambda t)^{7/3}d \;= \frac{2}{3\lambda}t\, g\sin\theta\int (R_o + \lambda t)^{7/3}d(R_o + \lambda t)$$

Που οδηγεί στην

$$\omega R^{10/3} = \frac{g\sin\theta}{5}\left\{\left[R_o + \lambda t\right]^{10/3} - R_o^{10/3}\right\} = \frac{g\sin\theta}{5}\left\{R^{10/3} - R_o^{10/3}\right\}$$

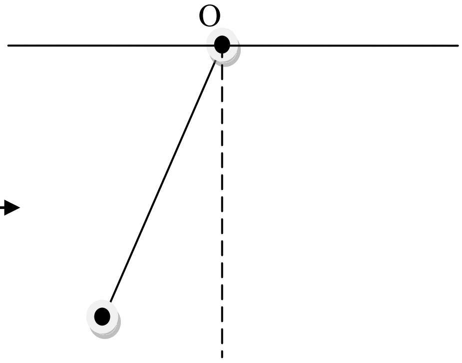
Τελικά έχουμε ότι

$$\omega = \frac{g\sin\theta}{5}\left\{1 - \left(\frac{R_o}{R}\right)^{10/3}\right\}$$

4ο. Θέμα **Μόρια 25**

a. Εξηγείστε πώς μεταβάλλονται οι καμπύλες των σχημάτων 23-2, 23-3 ανάλογα με τον συντελεστή ποιότητας Q. Υπόδειξη: Θεωρείστε ότι τα σχήματα 23-2, 23-3 προκύπτουν για m=1kg και Q=5.5 και παρουσιάστε τις καμπύλες για Q=1,3,10,30. Ποια ή μορφή τους στην ιδεατή περίπτωση όπου δεν υπάρχουν τριβές; (5)

b. Θεωρείστε ότι μία σημειακή μάζα m αναρτάται από σημείο Ο μέσω ενός νήματος μήκους l (βλ. σχήμα). Θεωρείστε ότι οι απομακρύνσεις πολύ μικρές έτσι ώστε πρακτικά η μάζα m να κινείται κατά τον άξονα x.



i. Δείξτε ότι η μάζα m εκτελεί απλή αρμονική ταλάντωση με συχνότητα $\omega_0 = \sqrt{g/l}$. (5)

ii. Θεωρείστε τώρα ότι το σημείο ανάρτησης του νήματος Ο εκτελεί απλή αρμονική ταλάντωση κατά τη διεύθυνση x $x_O = \xi_0 \cos(\omega t)$. Βρείτε την εξίσωση κίνησης της μάζας m χρησιμοποιώντας α) αδρανειακό και β) μη αδρανειακό παρατηρητή που βρίσκεται στο Ο. (5)

iii. Για το αδρανειακό σύστημα, βρείτε στη σταθερή κατάσταση το πλάτος ταλάντωσης της μάζας και τη διαφορά φάσης που έχει με την κίνηση του σημείου ανάρτησης (5)

iv. Δείξτε ότι όταν η συχνότητα ω είναι αρκετά μικρότερη από τη συχνότητα συντονισμού η τροχιά της μάζας είναι όπως η τροχιά που ακολουθεί κατά την ελεύθερη ταλάντωση όπου το μήκος του νήματος όμως είναι μεγαλύτερο από το πραγματικό. Αντίστοιχα όταν η συχνότητα είναι αρκετά μεγαλύτερη από τη συχνότητα συντονισμού η τροχιά της μάζας αντιστοιχεί στην τροχιά της ελεύθερης ταλάντωσης με μήκος μικρότερο του πραγματικού. (5)

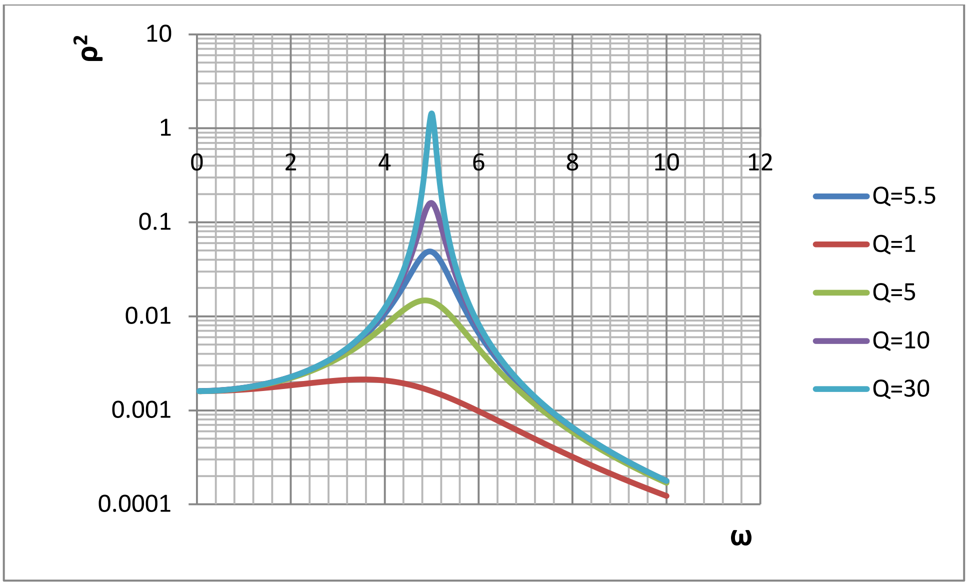
Θεωρείστε ότι δεν υπάρχουν τριβές.

<u>ΑΠΑΝΤΗΣΗ</u>

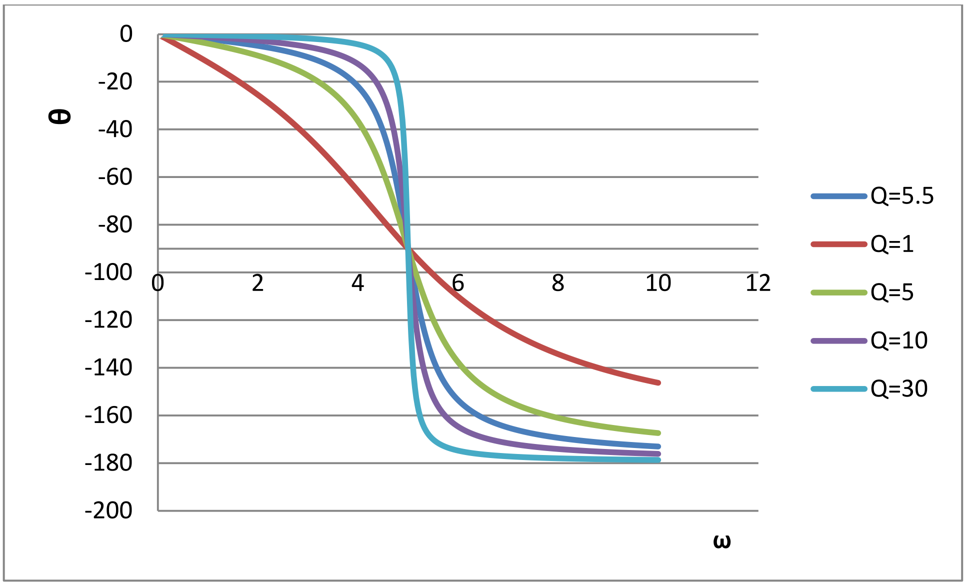
(α) Καταρχάς οι σχέσεις που αντιστοιχούν στα δύο σχήματα είναι οι 23.11 και 23.12 οι οποίες συναρτήσει του συντελεστή ποιότητας $Q = \dfrac{\omega_0}{\gamma}$ γράφονται ως εξής:

$$\rho^2 = \frac{1}{m^2\left[\left(\omega^2-\omega_0^2\right)+\gamma^2\omega^2\right]} = \frac{1}{m^2\left[\left(\omega^2-\omega_0^2\right)+\left(\dfrac{\omega_0\omega}{Q}\right)^2\right]}$$

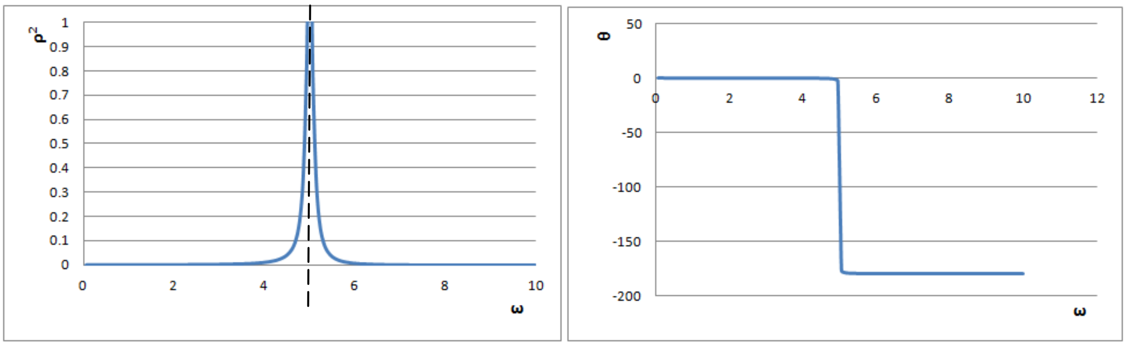
με αντίστοιχη γραφική παράσταση:



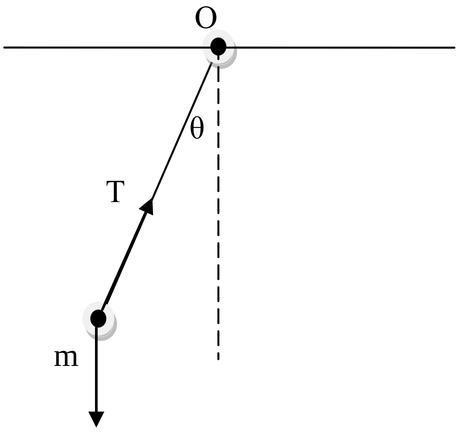
και $\tan\theta = -\frac{\gamma\omega}{\omega_0^2-\omega^2} = -\frac{\omega_0\omega/Q}{\omega_0^2-\omega^2}$ με αντίστοιχη γραφική παράσταση:



Στην περίπτωση χωρίς απόσβεση τα αντίστοιχα σχήματα είναι τα παρακάτω



(β) (i) Από την ανάλυση της τάσης του νήματος έχουμε για την κατακόρυφη διεύθυνση $T\cos\theta = mg$. Επομένως η δύναμη που ασκείται στην διεύθυνση x είναι



$$F_x = T\sin\theta = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta \approx mg\frac{x}{l}$$

Ο νόμος του Νεύτωνα δίνει επομένως $m\frac{d^2x}{dt^2} = -mg\frac{x}{l}$. Είναι όπως η εξίσωση του ελατηρίου (βιβλίο εξ. 21.2) με $k = \frac{mg}{l}$. Επομένως θα έχουμε ότι η συχνότητα είναι

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{l}}$$

(ii) Η εξίσωση του Νεύτωνα για αδρανειακό παρατηρητή είναι:

$$m\frac{d^2x}{dt^2} = -mg\frac{\left(x - x_O\right)}{l}$$

Για μη αδρανειακό παρατηρητή που βρίσκεται στο σημείο Ο η θέση της μάζας m είναι η σχετική θέση $x_{m,O}$. Επειδή το σύστημα του παρατηρητή επιταχύνεται με επιτάχυνση $\ddot{x}_O = -\xi_0\omega^2\cos\left(\omega t\right)$ ο νόμος του Νεύτωνα για τη μάζα m θα συμπεριλάβει τις ψευδοδυνάμεις και θα είναι:

$$m\frac{d^2x_{m,O}}{dt^2} = -mg\frac{x_{m,O}}{l} + m\xi_0\omega^2\cos\left(\omega t\right) \Rightarrow m\frac{d^2x}{dt^2} - m\frac{d^2x_O}{dt^2} = -mg\frac{x_{m,O}}{l} + m\xi_0\omega^2\cos\left(\omega t\right) \Rightarrow$$

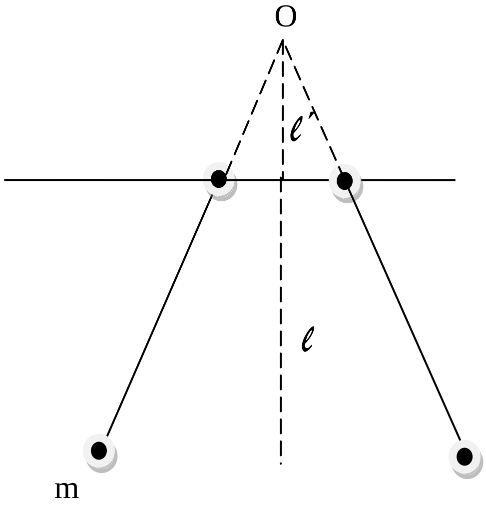
$$m\frac{d^2x}{dt^2} + m\xi_0\omega^2\cos\left(\omega t\right) = -mg\frac{x_{m,O}}{l} + m\xi_0\omega^2\cos\left(\omega t\right) \Rightarrow m\frac{d^2x}{dt^2} = -mg\frac{\left(x - x_O\right)}{l}$$

Δηλαδή καταλήγουμε στην ίδια σχέση με τη προηγούμενη περίπτωση.

(iii) Έχουμε από την εξίσωση κίνησης $m\frac{d^2x}{dt^2} = -mg\frac{\left(x - x_O\right)}{l} \Rightarrow m\frac{d^2x}{dt^2} = -mg\frac{x}{l} + mg\frac{\xi_0}{l}\cos\omega t$ δηλαδή η εξίσωση της ταλάντωσης με απόσβεση (βιβλίο σχέση 21.8) όπου $k = \frac{mg}{l}$ και $F_0 = \frac{mg\xi_0}{l}$. Επομένως η λύση σταθερής κατάστασης είναι η (βιβλίο) 21.10,21.12 δηλαδή $x = \frac{F_0}{m\left(\omega_0^2 - \omega^2\right)}\cos\omega t = \frac{\frac{g\xi_0}{l}}{\left(\omega_0^2 - \omega^2\right)}\cos\omega t$

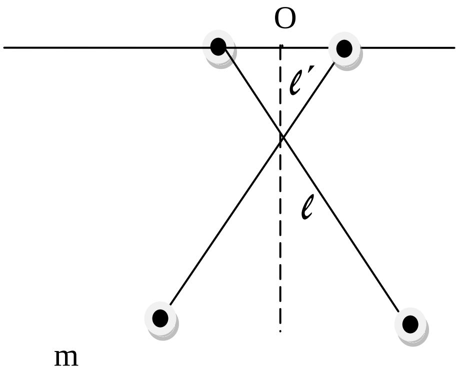
Η διαφορά φάσης θα είναι 0 ή -π ανάλογα με την συχνότητα ω. Αν ω>ω_0 τότε η διαφορά φάσης είναι –π (βλ. προηγούμενο ερώτημα 4^{ου} θέματος)

(iv) Αν ω<ω$_0$ $x = C\cos\omega t$ (C>0) και η μάζα κινείται στην ίδια κατεύθυνση με το σημείο Ο. Όπως φαίνεται στο σχήμα αυτό αντιστοιχεί σε μεγαλύτερο μήκος του νήματος. Δεδομένου ότι η θέση της μάζας είναι $\left(\frac{\frac{g\xi_0}{l}}{\left(\omega_0^2-\omega^2\right)}\cos\omega t, 0\right)$ και του σημείου Ο $\left(\xi_0\cos\omega t, 0\right)$ μπορούμε να υπολογίσουμε το «φαινόμενο μήκος» παίρνοντας πχ τις θέσεις μέγιστης απομάκρυνσης. Τότε από τα όμοια τρίγωνα θα έχουμε



$$\frac{\frac{\omega_0^2\xi_0}{\left(\omega_0^2-\omega^2\right)}}{l+l'} = \frac{\xi_0}{l'} \Rightarrow l' = l\frac{\omega_0^2-\omega^2}{\omega^2} \Rightarrow l+l' = l\left(\frac{\omega_0}{\omega}\right)^2$$

Αντίστοιχα στην περίπτωση όπου ω>ω$_0$ τότε η διαφορά φάσης είναι π και θα έχουμε το διπλανό σχήμα. Με ανάλογο τρόπο έχουμε ότι $l - l' = l\left(\frac{\omega_0}{\omega}\right)^2$



What Drives Glacial Cycles?

Massive reorganizations of the ocean-atmosphere system, the authors argue, are the key events that link cyclic changes in the earth's orbit to the advance and retreat of ice sheets

by Wallace S. Broecker and George H. Denton

Eight times within the past million years, something in the earth's climatic equation has changed, allowing snow in the mountains and the northern latitudes to remain where it had previously melted away. The snow compacted into ice, and the ice built up into glaciers and ice sheets. Over tens of thousands of years, the ice sheets reached thicknesses of several kilometers; they planed, scoured and scarred the landscape as far south as central Europe and the midwestern U.S. And then each glacial cycle came to an abrupt end. Within a few thousand years, the ice sheets shrank back to their present-day configurations.

Over the past 30 years, evidence has mounted that these glacial cycles are ultimately driven by astronomical factors: slow, cyclic changes in the eccentricity of the earth's orbit and in the tilt and orientation of its spin axis. By altering the intensity of the seasons, the astronomical cycles somehow tip the balance between glacial buildup and glacial retreat. But what is the link between astronomy and the ice ages? How are the seasonality changes leveraged into global changes in climate?

Any answer must contend with the vast array of evidence that has accumulated about the nature, timing and extent of the climatic shifts that accompanied ice buildup and retreat. Many workers have proposed that the seasonality changes act directly on the ice sheets of the Northern Hemisphere. A reduction in summer sunshine allows ice to build up, and an increase melts it away; the ice in turn alters the earth's climate. In contrast, we think the ice sheets were a consequence of broader climatic events. By altering patterns of evaporation and rainfall, the changes in seasonal intensity appear to have caused the ocean and atmosphere (a single, coupled system) to flip from one mode of operation to another, very different mode. With each flip, ocean circulation changed and heat was carried around the globe differently, the properties of the atmosphere were altered, climate changed—and the ice sheets grew or shrank.

ICE FIELD IN PATAGONIA ends in a deep glacial lake. Such Southern Hemisphere glaciers have grown and shrunk in concert with the great northern ice sheets, according to radiocarbon dating of vegetation (such as the trees in the foreground) that was overwhelmed by advancing glaciers or that took root after their retreat. The timing is a puzzle because the intensity of summer sunshine, which is thought to influence ice growth, changes on quite different schedules at middle latitudes in the two hemispheres.

Our proposal is not a rejection of the astronomical theory of the ice ages but an extension of it. The hypothesis was first proposed in 1842, just a few years after the Swiss-American naturalist Louis Agassiz argued that polished and scarred rocks and heaps of detritus in the Alps recorded some past age of glaciers. In that year the French mathematician Joseph A. Adhémar suggested that astronomically driven changes in the intensity of the seasons might periodically trigger glaciation.

The Yugoslav astronomer Milutin Milankovitch refined and formalized the hypothesis in the 1920's and 1930's. The astronomical pacemaker he advocated has three components, two that change the intensity of the seasons and a third that affects the interaction between the two driving factors. The first is the tilt of the earth's spin axis. Currently about 23.5 degrees from the vertical, it fluctuates from 21.5 degrees to 24.5 degrees and back every 41,000 years. The greater the tilt is, the more intense seasons in both hemispheres become: summers get hotter and winters colder.

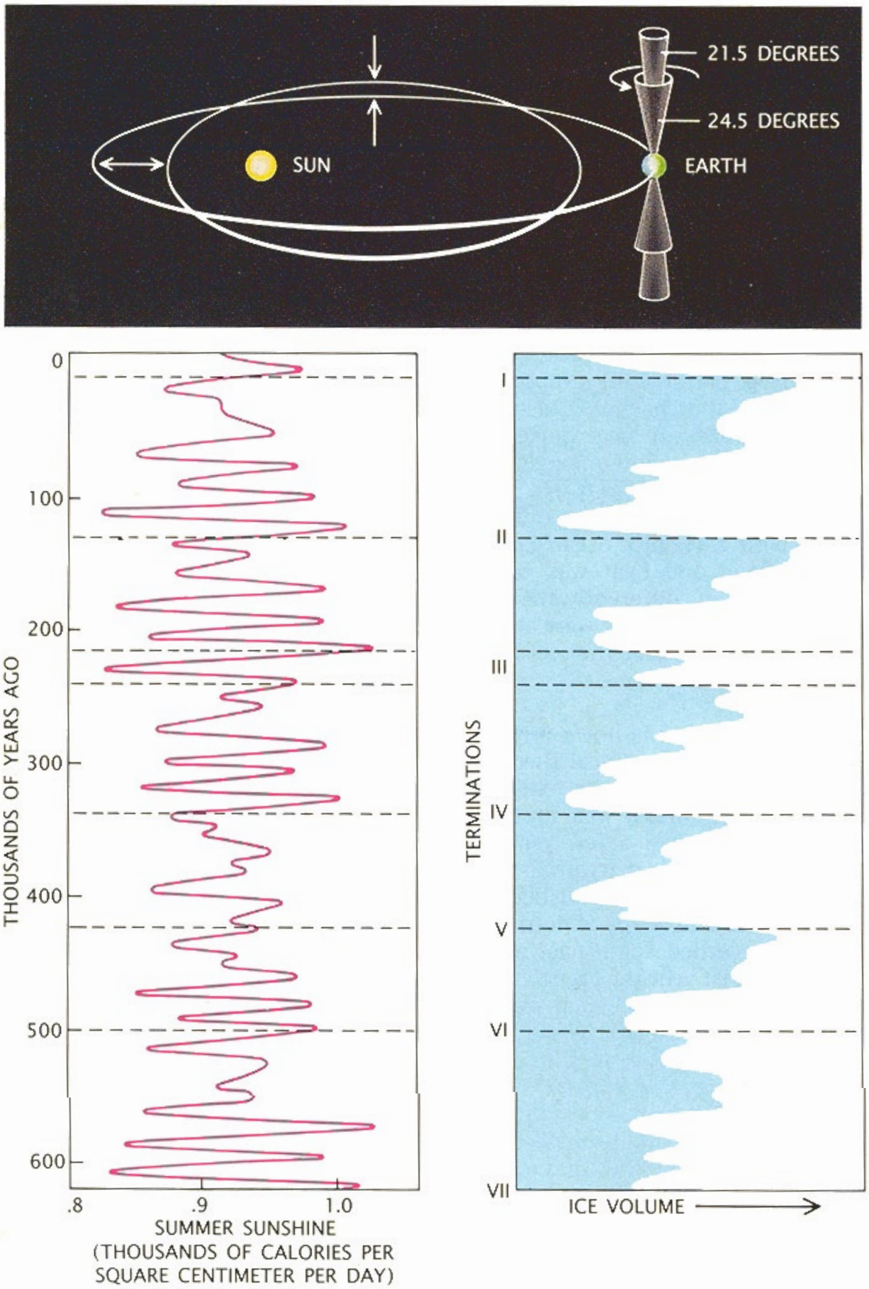
The second, weaker factor controlling seasonality is the shape of the earth's orbit. Over a period of 100,000 years, the orbit stretches into a more eccentric ellipse and then grows more nearly circular again. As the orbital eccentricity increases, the difference in the earth's distance from the sun at the orbit's nearest and farthest points grows, intensifying the seasons in one hemisphere and moderating them in the other. (At present the earth reaches its farthest point during the Southern Hemisphere winter; as a result, southern winters are a little colder—and summers a little warmer—than their northern counterparts.)

A third astronomical fluctuation governs the interplay between the tilt and eccentricity effects. It is the precession, or wobble, of the earth's spin axis, which traces out a complete circle on the background of stars about every 23,000 years. The precession determines whether summer in a given hemisphere falls at a near or a far point in the orbit—in other words, whether tilt seasonality is enhanced or weakened by distance seasonality. When these two controllers of seasonality reinforce each other in one hemisphere, they oppose each other in the opposite hemisphere.

WALLACE S. BROECKER and GEORGE H. DENTON bring diverse interests to their study of ice ages. Broecker got his Ph.D. at Columbia University in 1958 and has pursued his career there. He is now professor of geochemistry at the Lamont-Doherty Geological Observatory of Columbia University. In addition to ancient climates, he follows research interests in ocean chemistry, isotope dating and environmental science. Denton is professor of geology at the University of Maine. After earning a Ph.D. at Yale University in 1965, he did postdoctoral work at the University of Stockholm and then moved to Maine. He has spent 36 seasons in the field studying the timing and extent of glacial advances, 22 of them in Antarctica and elsewhere in the Southern Hemisphere.

Milankovitch calculated that these three factors work together to vary the amount of sunshine reaching the high northern latitudes in summer over a range of some 20 percent—enough, he argued, to allow the great ice sheets that advanced across the northern continents to grow during intervals of cool summers and mild winters. For many years, however, the lack of an independent record of ice-age timing made the hypothesis untestable.

In the early 1950's Cesare Emiliani, working in Harold C. Urey's laboratory at the University of Chicago, produced the first complete record of the waxings and wanings of past glaciations. It came from a seemingly odd place, the sea floor. Single-cell marine organisms called foraminifera house themselves in shells made of calcium carbonate. When the foraminifera die, sink to the bottom and contribute to the sea-floor sediments, the carbonate of their shells preserves certain characteristics of the seawater they inhabited. In particular, the ratio of a heavy isotope of oxygen (oxygen 18) to ordinary oxygen (oxygen 16) in the carbonate preserves the ratio of the two oxygens in the water molecules.

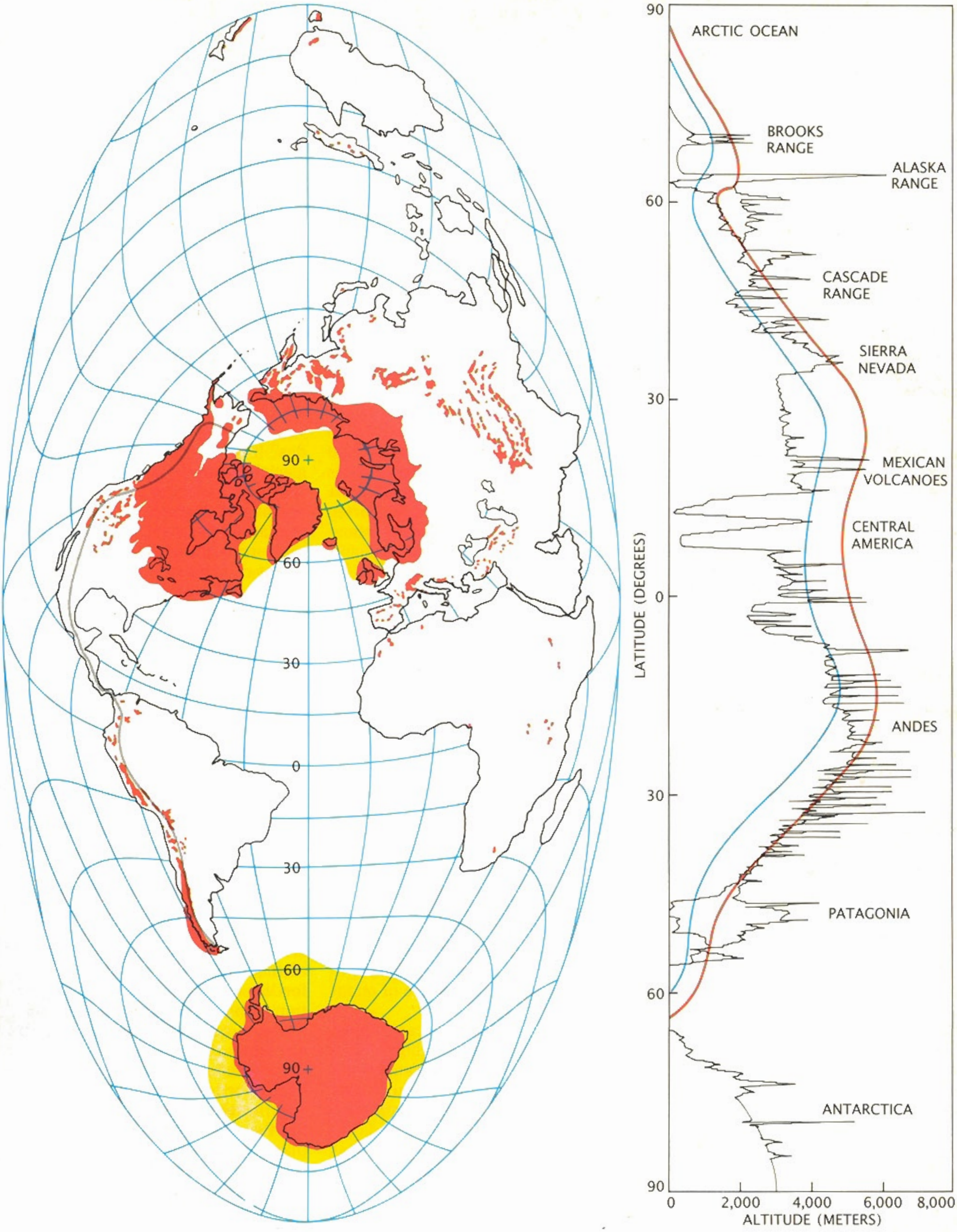


ASTRONOMICAL CYCLES (*top*) **are the pacemaker of glaciation. The cycles—23,000 to 100,000 years in length—affect the eccentricity of the earth's orbit, the orientation of its spin axis (which slowly traces out a cone in space) and the tilt of the axis (which affects the width of the cone). The effect of the changes on the intensity of summer sunshine at high northern latitudes is shown at the left. The curve at the right indicates the volume of the earth's ice sheets, determined from isotopic studies of sea-floor sediments. Ice volume climbs gradually for about 100,000 years and then falls abruptly in ice-age terminations that correspond to episodes of increasing summer sunshine at northern latitudes. (Seasonality varies differently in the Southern Hemisphere, which suggests that northern seasonality must be what drives ice ages.)**

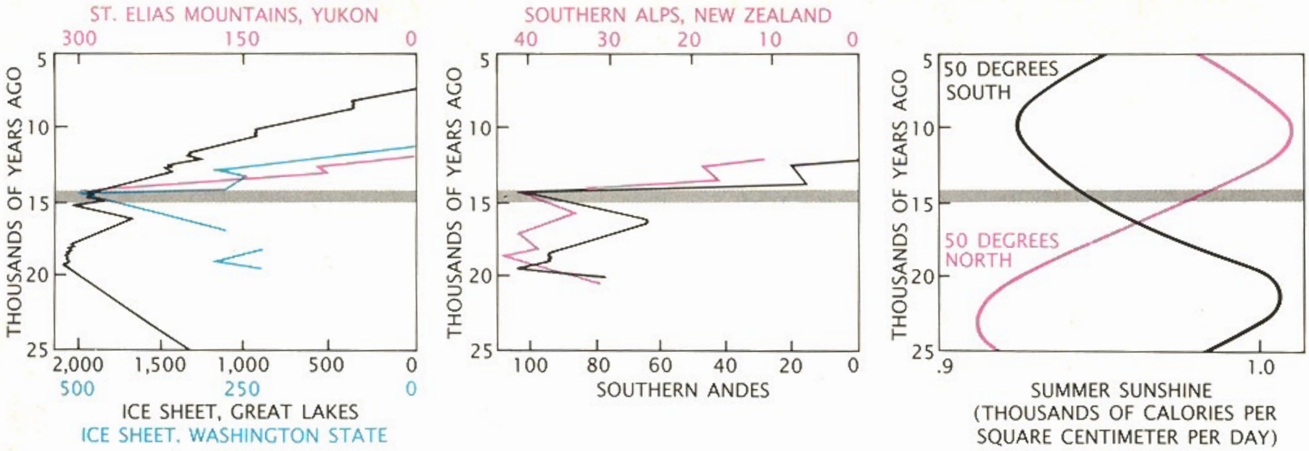
It is now understood that the ratio of oxygen isotopes in seawater closely tracks the proportion of the world's water that is locked up in glaciers and ice sheets. A kind of meteorological distillation accounts for the link. Water molecules containing the heavier isotope tend to condense and fall as precipitation a tiny bit more readily than molecules containing the lighter isotope. Hence, as water vapor evaporated from warm oceans moves away from the source, its oxygen 18 preferentially returns to the oceans in precipitation. What ultimately falls as snow on ice sheets and mountain glaciers is relatively depleted of oxygen 18. As the oxygen 18-poor ice builds up, the oceans become relatively enriched in the isotope. The larger the ice sheets grow, the higher the proportion of oxygen 18 becomes in seawater—and hence in the sediments.

Analyzing cores drilled from sea-floor sediments, Emiliani found that the isotopic ratio rose and fell in rough accord with the cycles Milankovitch had predicted. Since that pioneering observation, oxygen-isotope measurements have been made on hundreds of cores. A chronology for the combined record enabled James D. Hays of Columbia University, John Imbrie of Brown University and Nicholas Shackleton of the University of Cambridge to show in 1976 that the record contains the very same periodicities as the orbital processes.

Over the past 800,000 years, the global ice volume has peaked every 100,000 years, matching the period of the eccentricity variation. In addition, "wrinkles" superposed on each cycle—small decreases or surges in ice volume—have come at intervals of roughly 23,000 and 41,000 years, in keeping with the precession and tilt frequencies. Imbrie, working with a group called SPECMAP, later strengthened the case for the astronomical theory even more when he showed that the amplitude of the shorter-period signals has varied exactly as one would expect if the signals were be-



ICE SHEETS AND MOUNTAIN GLACIERS expanded in both hemispheres during the last ice age. The map (an unusual equal-area projection) shows the extent of land ice (*red*) and sea ice (*yellow*) on all the continents at peak glaciation some 19,500 years ago. (Land ice extended beyond some present coastlines because the sea level was lower.) The graph traces the average elevation of mountain snow lines on the American cordillera, plotted along the north-south transect indicated on the map. Ice-age snow lines (*blue line*) were about 1,000 meters lower than snow lines are today (*red*), regardless of latitude.



TIMING of glacial retreat was identical in the Northern Hemisphere (*left*) and in the Southern Hemisphere (*center*). The graphs give the extent of mountain glaciers and ice sheets from their source regions (in kilometers) and show that in every case dramatic retreat began 14,000 years ago. Changes in seasonal intensity could not have driven the retreat directly, because even though northern summers were getting stronger, summers in the Southern Hemisphere were weakening (*right*).

ing modulated by distance seasonality.

To be sure, there were loose ends. The 100,000-year variation has a much weaker effect on seasonal sunshine than the shorter cycles do, and yet it apparently sets the fundamental frequency of glaciation. The shorter cycles emerge only in the wrinkles in the isotopic record. What is more, the calculated seasonality cycles rise and fall smoothly, but the ice curve is sawtoothed: the ice grows episodically for nearly 100,000 years and then crashes in a few thousand, in a period of strengthening northern summers.

Workers have sought answers to both puzzles in the physics of the ice sheets and the underlying rock, which sinks under the weight of the ice. For example, William R. Peltier and William T. Hyde of the University of Toronto have built a theoretical model that incorporates assumptions about how the bedrock sinks and that closely reproduces both the dominance of the 100,000-year cycle and the rapid retreat of the ice. In the model, it takes nearly 100,000 years for an ice sheet to reach a critical size, at which point the ductile rock below the earth's crust begins to flow rapidly and allows the burdened crust to sink. The surface of the ice sheet drops; warmed by the lower elevation, the ice can melt rapidly when the shorter-period cycles bring the next episode of strong northern summers.

Peltier and Hyde's model, like many other models, assumes that Northern Hemisphere seasonality changes drive glacial advance and retreat directly, with bedrock response shaping each cycle and setting its length. Yet the assumption suffers a crucial problem: glaciers grew and retreated in the Southern Hemisphere as well. Studies by geologists, including the late John H. Mercer of Ohio State University and Stephen C. Porter of the University of Washington, show that during the last ice age, climate changed at the same times and by comparable amounts in the middle latitudes of the Southern Hemisphere—even though seasonality there varies on a quite different schedule.

They and others have found, for example, that during the last ice age the earth's mountain glaciers also expanded. The evidence—from the heaps of debris plowed up by the glaciers, known as moraines—is as clear in the tropics (New Guinea, Hawaii, Colombia and East Africa) and the southern temperate latitudes (Chile, Tasmania and New Zealand) as it is in northern temperate latitudes (the Cascades, the Alps and the Himalayas). On all the mountains studied so far, regardless of geographic setting or precipitation rate, the snow line descended by about one kilometer, corresponding to a drop in temperature of about five degrees Celsius.

Where organic material was trapped in the moraines, radiocarbon dating shows that the glaciers advanced and retreated on the same schedule. They fluctuated near their maximum extent between about 19,500 and 14,000 years ago, about the same time as the glaciation of northern continents peaked. Then, just as the northern ice sheets began to shrink, the mountain glaciers underwent a dramatic retreat that sharply reduced their size by about 12,500 years ago.

How could changes in summer sunshine at the latitude of Iceland have caused glaciers to grow and retreat in New Zealand and the southern Andes? If orbital cycles do indeed drive glacial cycles by acting directly on northern ice sheets, the response to seasonality changes in the high northern latitudes must be strong enough to override the effects of the very different changes in the Southern Hemisphere. One possibility is that the northern ice sheets themselves translate Northern Hemisphere seasonality into climatic change around the world.

Two links between the northern ice sheets and ice growth worldwide have been proposed, but neither one bears up well under scrutiny. One invokes sea level, which would have dropped as the growth of the northern ice locked up much of the world's water. Since glaciers can grow only on land, the drop in sea level might have allowed southern glaciers to expand onto the exposed continental shelves even without a global change in temperature. Later, when the northern ice sheets melted, the rise in sea level might have broken up the margins of the Southern Hemisphere glaciers, forcing them to retreat. The explanation is plausible only for Antarctica, however, because most mountain glaciers do not approach the sea.

The second proposal relies on the high albedo, or reflectivity, of the vast northern ice sheets. By reducing the absorption of sunlight by the planet as a whole, the ice might have led to global cooling and allowed glaciers to grow at southern latitudes. Yet computer climate models show that the albedo effects of Northern Hemisphere ice sheets should be confined to northern latitudes. Also, if ice albedo does drive global climatic change, one would expect to find a pronounced north-to-south gradient in the mountain-glacier record, with mountains adjacent to the northern ice sheets recording the greatest snow-line lowering and the Andes, say, showing very little change. No such gradient is seen.

Any causal link between the ice sheets and global climatic change also must contend with the timing of the mountain-glacier retreat. Both the northern ice sheets and the mountain glaciers began their retreat from the last glacial maximum at the same time, about 14,000 years ago. The continental glaciers took about 7,000 years to melt away, whereas the mountain glaciers shrank much more quickly. The disparity suggests that the northern ice sheets cannot be calling the tune for climate over the rest of the earth.

If the ice sheets themselves cannot link the astronomical cycles to the climatic shifts, what can? Clues come from core samples drilled from depths of as much as two kilometers in the ice that still blankets Greenland and Antarctica. The first thing the ice cores offer is confirmation of the global and synchronous character of ice-age climatic changes.

The oxygen 18 content of glacial ice is depleted in general, but the exact content records the local temperature at the time the ice was laid down. (The colder a parcel of air becomes, the more of its water vapor is likely to have fallen already in precipitation, reducing the oxygen 18 content of the remaining vapor.) Isotopic studies of the Greenland and Antarctic cores show that during the last glaciation both poles cooled—to as much as 10 degrees C below today's temperatures—and warmed in step.

The ice also revealed something much more intriguing. Groups led by Hans Oeschger of the University of Bern and Claude Lorius of the Laboratory of Glaciology and Geophysics of the Environment, near Grenoble, measured the carbon dioxide content of the tiny bubbles of ancient air trapped in the ice. They found that during the last glaciation the carbon dioxide concentration of the atmosphere was about two thirds of its interglacial level. The carbon dioxide curve pointed to a missing ingredient in the climatic recipe: the ocean.

Only a major shift in the ocean's operation could account for such a dramatic change in atmospheric composition. After all, the oceans hold 60 times as much carbon dioxide as the atmosphere; because the gas readily diffuses between the ocean surface and the atmosphere, its concentration in surface waters regulates the atmospheric concentration.

Living things in turn control the surface-water concentration, by acting as a biological pump that transfers carbon dioxide from the surface to the ocean depths. In the course of photosynthesis, the tiny green plants of the ocean's sunlit upper layers capture dissolved carbon dioxide to form organic tissue. Some of the plant matter, as well as animal tissue nourished by the plants, eventually sinks into the deep sea, where bacteria oxidize it back to carbon dioxide. Thus, the gas is continuously pumped into the abyss, together with nutrients such as phosphate and nitrate.

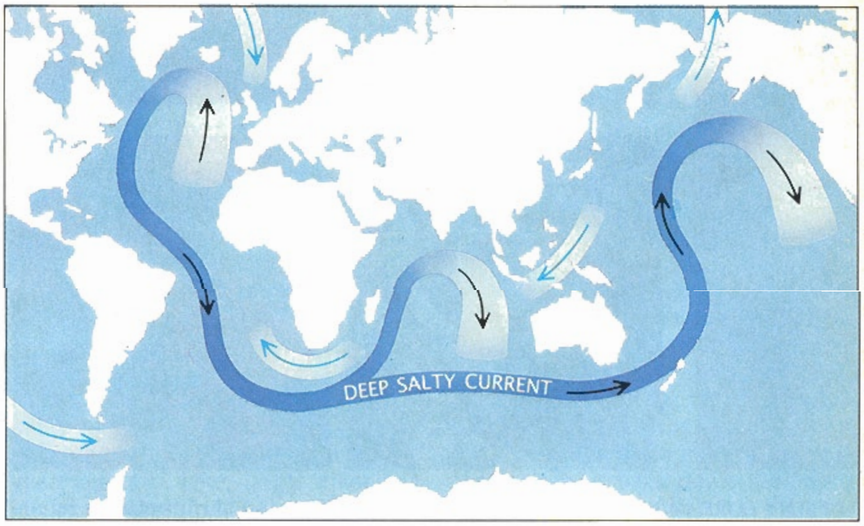
The efficiency of this pump depends not only on the surface community's population and species but also on vertical mixing patterns. The exact link between pumping efficiency and ocean circulation is controversial, but one can imagine, for example, that if the mixing of deep waters with the surface is slowed, surface plant life will have more time to deplete the shallow water of carbon dioxide before more of the gas is stirred up from the depths. During glacial time, some combination of altered mixing and changes in ecology must have made the biological pump more efficient.

The first indications that the ice-age ocean did operate differently came from fossil evidence: changes in the populations of microorganisms that inhabit water masses of specific temperature and salinity, studied by William F. Ruddiman and Andrew McIntyre of Columbia University and by Detmar F. Schnitker of the University of Maine. More recently a geochemical technique pioneered by Edward A. Boyle of the Massachusetts Institute of Technology provided dramatic and direct confirmation that the ocean circulated differently during the last glaciation.

Boyle discovered that, for unknown reasons, the distribution of cadmium in today's oceans closely matches that of phosphate and nitrate nutrients. Because the cadmium ion has the same charge and size as calcium, Boyle guessed that cadmium might substitute for calcium in the calcium carbonate of foraminiferal shells. If it does, measurements of cadmium in shells from sediment cores might reveal the distribution of nitrate and phosphate in the glacial ocean.

Boyle's intuition proved correct when he found that foraminifera in the present-day ocean do incorporate cadmium in a constant proportion to its abundance in seawater. He then measured cadmium in sediment cores. The result was exciting: a key signature of the Atlantic's present-day circulation was missing during glacial time, until about 14,000 years ago.

Currently the Atlantic's deep water contains only about half as much phosphate and nitrate as the deep waters of the Pacific and Indian oceans. The



DEEP SALTY CURRENT threads the world's oceans, compensating for the transport of water vapor by the atmosphere. (Light blue arrows indicate shallow return flow.) The current originates in the North Atlantic, where northward-flowing warm water that is unusually saline (and therefore dense) because of excess evaporation is chilled, which increases its density further. It sinks into the abyss and flows southward, out of the Atlantic. Most of the salty water that is supplied by this Atlantic "conveyor" mixes upward in the Pacific, making up for excess precipitation there. The Atlantic conveyor—and probably the entire system—was disrupted during glacial time.

low nutrient content reflects the water's recent sojourn near the surface (where biological activity depletes the nutrients). Every winter at about the latitude of Iceland, water of relatively high salinity, flowing northward at intermediate depths (perhaps 800 meters), rises as winds sweep the surface waters aside. Exposed to the chill air, the water releases heat, cooling from 10 degrees C to two degrees. The water's high salinity together with the drop in temperature makes it unusually dense, and it sinks again, this time all the way to the ocean bottom.

The formation of the North Atlantic deep water, as it is called, gives off a staggering amount of heat. Equal to about 30 percent of the yearly direct input of solar energy to the surface of the northern Atlantic, this bonus of heat accounts for the surprisingly mild winters of Western Europe. (The warming is often mistakenly ascribed to the Gulf Stream, which ends well to the south.) The magnitude of the vertical circulation is also immense, averaging 20 times the combined flow of all the world's rivers. Indeed, much of the deep water in the world's oceans ultimately originates here. From its source the water floods the deep Atlantic, curves around the southern tip of Africa and joins the deep current that circles Antarctica and distributes deep water to the other oceans.

As the deep water ages and travels away from the site of its formation, it collects sinking phosphate and nitrate, which results in a gradient of increasing nutrient levels. By measuring the cadmium content of foraminifera that lived near the bottom, Boyle found that during glacial time the nutrients were more uniformly distributed through the depths of the world's oceans. In addition, the concentration in the glacial Atlantic peaked in the deepest parts rather than at intermediate depths, as it does today.

These results bore out the implication of the earlier microfossil studies. The Atlantic "conveyor," which releases vast quantities of heat to the North Atlantic and sends immense volumes of water into the abyss, was shut down until the last ice age ended 14,000 years ago. In the absence of this key component, worldwide ocean circulation must have looked very different.

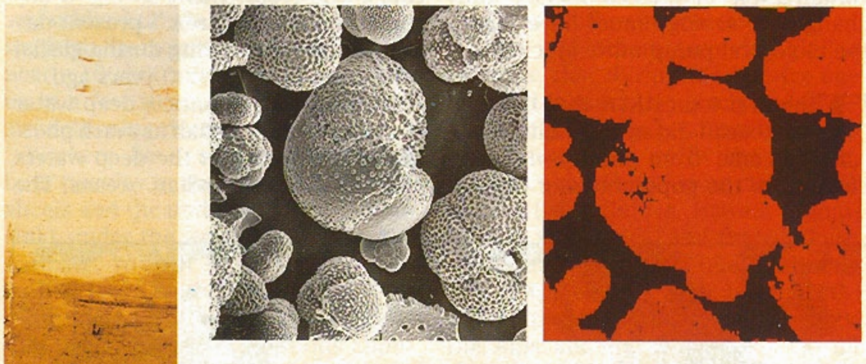
The sea and land evidence together points to a simultaneous change in the operation of the ocean and the atmosphere 14,000 years ago. The pattern of ocean circulation shifted dramatically; glaciers in both hemispheres began retreating, signaling global warming; and the carbon dioxide content of the atmosphere started to rise to interglacial levels. We think these events indicate a major reorganization of the joint ocean-atmosphere system—a jump from a glacial mode of operation to an interglacial mode. Indeed, we believe that abrupt jumps among several ocean-atmosphere modes may underlie glacial cycles in general.

We propose that changes in seasonality are the ultimate causes of these mode shifts. Although we can suggest no simple mechanisms linking seasonality, the ocean-atmosphere system and global climate, we can offer some insights.

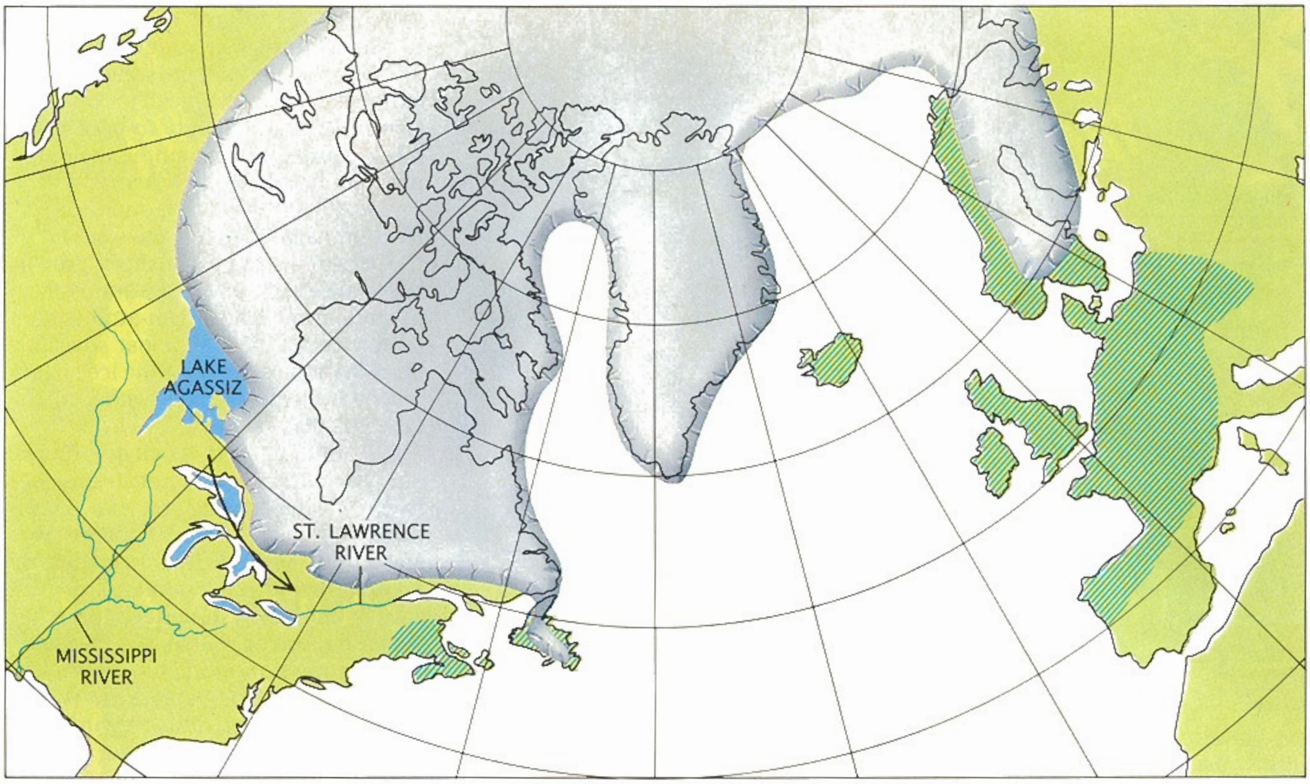
The atmosphere, which would certainly feel the effects of seasonality changes, strongly influences the circulation of the ocean. The link involves the distribution of salt. Prevailing winds transfer water evaporated from one part of the ocean to another region, where it falls as precipitation. The transport of vapor leaves a heritage of salt in the first region and dilutes the salinity of the second.

Now, the tendency of surface waters to sink into the depths and initiate a vertical conveyor belt like that of the North Atlantic depends on their density. Density reflects both temperature and salinity, but salinity is the decisive factor. (Surface water cools almost to the freezing point throughout the high latitudes in winter, but only where it is unusually saline does it sink into the abyss.) The system has a built-in nonlinearity: a gradual shift in atmospheric circulation, by changing salinity in regions such as the North Atlantic, could dramatically alter the global circulation pattern. Indeed, the Atlantic conveyor appears to be the most vulnerable part of the system, which may explain why it is Northern Hemisphere seasonality that drives global climatic changes.

A climatic event called the Younger Dryas, which took place several thousand years after the glaciers started to retreat, provides a smoking gun for this part of our case. It vividly illustrates the link between the transport of fresh water—in this case liquid water and not vapor—and ocean circulation. About 11,000 years ago the re-



SEDIMENT CORE (*left*) from the North Atlantic testifies to an abrupt change in circulation at the end of the penultimate glaciation about 128,000 years ago. The transition (identified by Gerard C. Bond of Columbia University) spans a few millimeters and represents about 50 years. A scanning electron micrograph of coarse material from the dark sediments (*bottom*) reveals abundant rock fragments, rich in silicon (*blue in an X-ray map*), presumably dropped by melting icebergs. The light-colored sediments (*top*) include almost no rock and are made up mainly of shells, rich in calcium (*red*), from marine organisms that inhabit warm waters. (Shells in the dark sediments came from cold-water species.) The sudden revival of the Atlantic conveyor must have warmed the surface, eliminating icebergs and altering ecology.



DIVERSION OF MELTWATER during the retreat of the North American ice sheet some 11,000 years ago may explain the 1,000-year cold spell known as the Younger Dryas. Lake Agassiz, fed by meltwater, had been draining down the Mississippi River to the Gulf of Mexico. When the retreat of the ice opened a channel to the east, however, the water flooded across the region of the Great Lakes to the St. Lawrence River (*arrow*). The influx of fresh water to the North Atlantic diluted the salinity of surface water, reducing its density and preventing it from sinking. The Atlantic conveyor was shut down: warm water could no longer flow northward, and a broad region around the North Atlantic was chilled (*hatched area*).

treat of the glaciers was well under way, and temperatures had risen to their interglacial levels. Suddenly, in as little as 100 years, northern Europe and northeastern North America reverted to glacial conditions. Pollen records show that the forests that had colonized postglacial Europe gave way to arctic grasses and shrubs (including the Dryas flower, for which the period is named), and the Greenland ice core records a local cooling of six degrees C. About 1,000 years later, this cold spell ended abruptly—in as little as 20 years, recent work by Willi Dansgaard of the University of Copenhagen suggests.

Boyle's cadmium measurements, together with the record of surface-water foraminifera in the North Atlantic, tell what happened. Both indicators return to their glacial state at the onset of the Younger Dryas. The conveyor belt had shut down once again. Deep-water formation had stopped, and so the warm intermediate-depth water that supplies Europe's bonus of heat could no longer flow northward. The chill over the region was dispelled only when the conveyor began running again 1,000 years later.

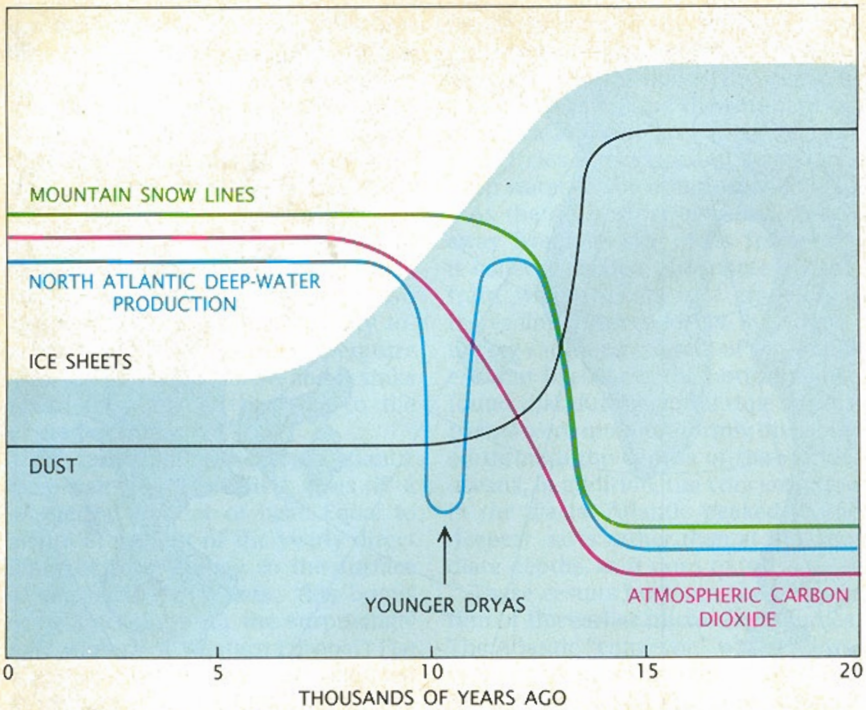
A massive influx of fresh water from the melting North American ice sheet seems to have killed the conveyor and precipitated the Younger Dryas. The ice sheet started shrinking 14,000 years ago; for the 7,000 years it took to melt away, it must have released fresh water at about the same rate as today's Amazon River. At first nearly all the meltwater from the southern edge of the massive ice sheet flowed down the Mississippi River to the Gulf of Mexico. About 11,000 years ago, however, a major diversion sent meltwater in torrents down the St. Lawrence River to the Atlantic.

A vast clearinghouse for meltwater, known as Lake Agassiz, had formed in the bedrock depression at the edge of the retreating ice sheet in what is now southern Manitoba. Until 11,000 years ago the lake, larger than any of the existing Great Lakes, had overflowed a bedrock lip to the south and drained down the Mississippi.. Then the retreat of the ice opened a channel to the east. The water level in Lake Agassiz dropped by 40 meters as water flowed across the region of the Great Lakes and down the St. Lawrence.

Foraminifera from surface waters of the Gulf of Mexico record this diversion. Their oxygen 18 content had been anomalously low, reflecting the oxygen 16-rich meltwater discharging from the Mississippi. About 11,000 years ago the isotopic ratio increased abruptly as the Lake Agassiz diversion shut off the meltwater flow to the Gulf.

The meltwater, meanwhile, poured into the North Atlantic close to the site of deep-water formation. There it reduced the salinity of surface waters (and hence their density) by so much that, in spite of severe winter cooling, they could not sink into the abyss. The conveyor belt stayed off until 1,000 years later, when a lobe of ice advanced across the western end of the Lake Superior basin and once again blocked the exit to the east. Lake Agassiz rose again by 40 meters, diverting the meltwater back down the Mississippi. The conveyor belt was reactivated, and Europe warmed up again.

The Younger Dryas links freshwater flow, ocean circulation and climate—but only regional climate. Only around the North Atlantic did the episode bring a sharp cooling; elsewhere its effects were slight



END OF THE LAST ICE AGE brought global changes, summarized here, that began at the same time about 14,000 years ago even though they proceeded at different rates. The circulation of the North Atlantic shifted abruptly from glacial to interglacial conditions (with a brief relapse during the Younger Dryas cold snap) as deep-water production resumed. At the same time, the amount of dust in the atmosphere dropped and the concentration of carbon dioxide started to increase. The shifts may have been part of a larger reorganization of the ocean and atmosphere that warmed the planet and caused mountain glaciers and ice sheets to start retreating.

or absent. Unlike the glaciations, the Younger Dryas affected only the transport of heat (from low latitudes to the North Atlantic) and not the global climate. How could a change in ocean-atmosphere operation during the ice ages have cooled the world as a whole?

The Greenland and Antarctic ice cores suggest part of an answer. The lower level of atmospheric carbon dioxide they record for the last glaciation would certainly have contributed to the cooling: carbon dioxide is a greenhouse gas that warms the earth's surface by trapping solar energy. Computer climate simulations suggest, however, that the global cooling caused by the observed drop in carbon dioxide would be at most two degrees C—less than half of what is recorded in the mountain glaciers.

Two other changes recorded in the ice cores must also have contributed. Ice-age air contains only half the postglacial level of methane. Methane, too, is a greenhouse gas, although the ice-age cooling attributable to reduced methane amounts to just a few tenths of a degree. In addition, dust is about 30 times as abundant in glacial-age ice as in more recent layers, confirming evidence from other sites that the ice-age atmosphere was exceedingly dusty. Dust, too, could have contributed to the cooling, by reflecting sunlight. Unfortunately, its effect is hard to quantify.

The dustiness and low methane content of the ice-age air do suggest that the glacial mode of ocean-atmosphere operation had imposed a dry climate. Dust, after all, blows from areas where vegetation is sparse, whereas methane is produced in swamps. Dry conditions (which are also recorded in ice-age landforms, such as sand dunes, and in pollen deposits) would have had their own effect on global temperatures. Temperature falls more rapidly with increasing altitude in a drier atmosphere; hence, the drying could have contributed to the depression of mountain snow lines.

Even added together, the impacts of carbon dioxide, methane, dust and drying may come up short in accounting for the temperature difference between the glacial and the interglacial planet. What else could have contributed? One possibility is that the ocean-atmosphere reorganization changed the characteristics of clouds and made them more reflective.

Clearly, our account of how changes in ocean-atmosphere operation could have cooled the planet is incomplete. Moreover, since we appeal to Northern Hemisphere seasonality to pace these mode shifts, we encounter the same problem faced by other theorists: Why is the 100,000-year astronomical cycle dominant when it is the weakest of the three? Perhaps ice-sheet growth has a feedback effect on atmospheric circulation. The ocean-atmosphere system might become most susceptible to a mode shift once the ice sheets reached a critical size—which might take 100,000 years.

Still, much recent evidence favors our basic proposal: transitions between glacial and interglacial conditions represent jumps between two stable but very different modes of ocean-atmosphere operation. If the earth's climate system does jump between quantized states, like the electrons around an atom, all climate indicators should register a transition simultaneously. In this regard, the evidence from the end of the last ice age is most impressive. The warming of North Atlantic surface waters, the onset of melting in the northern ice sheets and the mountain glaciers of the Andes, the reappearance of trees in Europe and changes in plankton ecology near Antarctica and in the South China Sea—all took place between 14,000 and 13,000 years ago.

If the global climate system does prove to have quantized states, climatologists will have gained new insight into the way astronomical forcing, acting mainly in high northern latitudes, could transform climate worldwide. They will also have new cause for concern over the earth's climatic future. Just as 14,000 years ago the earth was feeling the gradual forcing effect of stronger northern summers, so now it is subject to gradual forcing as human activity releases carbon dioxide and other greenhouse gases into the atmosphere. Will the climate system again respond abruptly, by flipping to an entirely new mode?

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